METHODOLOGY AND APPLICATION OF TWO-WAY ANOVA

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Abstract:

There are commonly two types of ANOVA tests for univariate analysis-One way ANOVA and Two-Way ANOVA. One way-ANOVA is used when one can interested in studying the effect of one independent variable factor on population, whereas Two-way ANOVA is used for studying the effects of two factors on population at the same time.

This paper describes the powerful statistical technique Two-way ANOVA that can be used in many applications. The application data were analyzed using Excel that performs these calculations.

Keywords: Two-way ANOVA test, normality tests, homoscedasticity tests, multiple comparison tests, EXCEL

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1. Introduction:

Two-Way ANOVA:

Using one-way ANOVA, we found out that the music treatment was helpful in improving the test results of our students. But this treatment was conducted on students of the same age group. What if the treatment was to affect different age group of students in different ways? or may be the treatment had varying effects depending upon the teacher who taught the class.

Moreover, how can we be sure as to which factors is affecting the results of the students more? May be the age group is more dominant factor responsible for a student's performance than the music treatment.

For such cases, when the outcome or dependent variable i.e. in our case the test score is affected by two independent variables or factors we use a slightly modified technique called Two-way ANOVA.

In the one-way ANOVA test, we found out that the group subjected to 'variable music' and 'no music at all' performed more or less equally. It means that the variable Music treatment did not have any significant effect on the students.

So while performing Two-way ANOVA test we will not consider the 'Variable Music' treatment for simplicity of calculation. Rather new factor age, will be introduce to find out how the treatment performs when applied to students of different age groups. This dataset looks like this

Class Group	Age Group(yrs)		
	4-8 yrs	8-13 yrs	13-17 yrs
A (Constant sound)	6	4	7
	5	5	6
	5	6	10
	2	9	8
	4	8	9
Sub-total A	22	32	40
B (No sound)	1	4	6
	3	5	8
	2	6	4
	1	7	7
	2	3	5
Sub-total B	9	25	30
Grand Total	31	57	70

Table 1

Here , there are Two factors- class group and age group with two and three levels respectively. So we now have six different groups of students based on different permutations of class groups and age groups and each different group has a sample size of 5 students.

A few questions that two-way ANOVA can answer about this dataset are:

1.   Do groups subjected to different music differ significantly in their test performance?

2.   Do students of different age differ significantly in their test performance?

3.   Is there a significant interaction between the factors? In other words, how do age and music interact with regards to a student's test performance? For example, it might be that younger students and elder students treated differently to such a music treatment.

4.   Can any differences in music and test performance be found in different age groups?

2. **Two Way ANOVA**

Implementation using a example.

Two-way ANOVA tells us about the Main effect and the interaction effect. The main effect is similar to a One –way ANOVA where the effect of music and age would be measured separately .Whereas the interaction effect is the one where both music and age are considered at the same time.

That's why a Two-way ANOVA can have up to three hypothesis, which are as follows:

Two null hypothesis will be tested if we have placed only one observation in each cell. For this example those hypothesis will be

H_1:All the music treatment group have equal mean score.

H_2:All the age groups have equal mean score.

For multiple observations in cells, we would also be testing a third hypothesis,

H_3: The factors are independent or the interaction effect does not exist.

F-statistic is compared for each hypothesis that we are testing.

Before we proceed with the calculation, have a look at the image below. It will help us better understanding the terms used in the formulas.

Table 2

		factor 2					
		Age Group(yrs)					
	Class Group	4-8 yrs	8-13 yrs	13-17 yrs	A_l	···········▶	factor 1 total
factor 1	A (Constant sound)	6,5,5,2,4 (22)	4,5,6,9,8 (32)	7,6,10,8,9 (40)	94		
	B (No sound)	1,3,2,1,2 (9)	4,5,6,7,3 (25)	6,8,4,7,5 (30)	64		
	A_m	31	57	70	316	···········▶	Grand Total (G)
		subtotal (A_{lm})					
	factor 2 total						

The table shown above is known as a contingency table. Here A1 represent the total of the sample based only on factor 1 and similarly Am represents the total of sample based only on factor 2.We will see in some time that these two are responsible for the main effect produced. Also a term A_{1m} is introduced which represents the subtotal of factor 1 and factor 2.This term will responsible for the interaction effect produced when both of the factors are considered at the same time . And we are already familiar with the Grand Total G, which is the sum of all the observations i.e. test score, irrespective of the factors.

Table 3

SUMMARY		4-8 yrs	8-13 yrs	13-17 yrs	Total
	A (Constant sound)				
Count		5	5	5	15
Sum		22	32	40	94
Average		4.4	6.4	8	6.26666667
Variance		2.3	4.3	2.5	4.92380952
	B (No sound)				
Count		5	5	5	15
Sum		9	25	30	64
Average		1.8	5	6	4.26666667
Variance		0.7	2.5	2.5	5.06666667
	Total				
Count		10	10	10	
Sum		31	57	70	
Average		3.1	5.7	7	
Variance		3.21111111	3.56666667	3.33333333	

We have calculated all the means-sound class mean, age group mean and mean of every group combination in the above table.

Now calculate the sum of square (SS) and degree of freedom (df) for sound class, age group and interaction between factor and levels.

Calculation of the Two-way ANOVA

$$\mathbf{df_{within}} = (r-1) * a * b = 4 * 2 * 3 = 24 \qquad SS_{within} =$$

$$\mathbf{MS_{within}} = \mathbf{SS_{within}} / \mathbf{df_{within}} = 59.2/24 = 2.46$$

$$\mathbf{SS_{sound}} = r.b.\sum_{i=1}^{2}(\bar{Y_\imath} - \bar{Y})^2 = 30$$

$$\mathbf{df_{sound}} = 2\text{-}1 = 1$$

$$\mathbf{SS_{age}} = r.a.\sum_{i=1}^{3}(\bar{Y_\imath} - \bar{Y})^2 = 78.86$$

$$\mathbf{df_{age}} = 3\text{-}1 = 2$$

In Two-way ANOVA, we also calculate SS and df which defines the combined effect of the two factors.

$$SS_{interaction} = r \times \sum_{i=1}^{2} \sum_{j=1}^{3} \left(\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...} \right)^2 = 1.8$$

$$df_{interaction} = (a - 1) \times (b - 1) = 2$$

$$MS_{interaction} = SS_{interaction} / df_{interaction} = 0.9$$

Since we have more than one source of variation (main effect and interaction effects, it is obvious that we will have more than one F-statistic also.

Now using these variances, we compute the value of F-statistic for main effect and interaction effects, so the value of F-statistic is,

F1=12.16 ,F2=15.98,F12=0.36

We cam see the critical values from the table,

F_{crit1}=4.25 ,F_{crit2}=3.40 ,F_{crit12}=3.40

If, for a particular effect, its F-value is greater than its F-critical value calculated using F-Table then we reject null hypothesis for that particular effect.

Steps to perform two-way ANOVA in Excel :

2.1 Steps to perform two-way ANOVA in Excel

Step 1: Click on the "Data" tab and then click "Data Analysis." If you don't see Data Analysis, load the 'Data Analysis Toolpak' using add-in option in MS-Excel.

Step 2: Click "ANOVA Two Factor with replication" and then click "OK.".The two-way ANOVA window will open

Step 3: Type the input range into the Input Range box. For example, if the data is in cells A1 to A25, type "A1:A25" into the box. Check the "Labels in first row" if we have column headers, and select the Rows radio button if the data is in rows.

Step 4: Type the numbers in the "row per sample" box.

Step 5: Select the output range. For example, click the "New Worksheet" radio button if we want output in new worksheet..

Step 6: Choose level of significance (α) . For most hypothesis tests, α=0.05 is standard one .

Step 7: Click "OK." To run two-way ANOVA. The data will be returned in your specified output range.

Step 8: Read the results to figure out if one can going to reject the null hypothesis or not.

1)   If the F-value is larger than the f-critical

2)   If the p-value is smaller than your alpha level

The results for two-way ANOVA test in our example look like

Table 4

	Age Group(yrs)				SUMMARY	4-8 yrs	8-13 yrs	13-17 yrs	Total		
Class Group	4-8 yrs	8-13 yrs	13-17 yrs		A (Constant sound)						
A (Constant sound)	6	4	7		Count	5	5	5	15		
	5	5	6		Sum	22	32	40	94		
	5	6	10		Average	4.4	6.4	8	6.26666667		
	2	9	8		Variance	2.3	4.3	2.5	4.92380952		
	4	8	9		B (No sound)						
B (No sound)	1	4	6		Count	5	5	5	15		
	3	5	8		Sum	9	25	30	64		
	2	6	4		Average	1.8	5	6	4.26666667		
	1	7	7		Variance	0.7	2.5	2.5	5.06666667		
	2	3	5		Total						
					Count	10	10	10			
					Sum	31	57	70			
					Average	3.1	5.7	7			
					Variance	3.21111111	3.56666667	3.33333333			
					ANOVA						
					Source of Variation	SS	df	MS	F	P-value	F crit
					Sample	30	1	30	12.1621622	0.001900531	4.25967727
					Columns	78.8666667	2	39.4333333	15.9864865	0.000038618	3.40282611
					Interaction	1.8	2	0.9	0.36486486	0.698075604	3.40282611
					Within	59.2	24	2.46666667			
					Total	169.866667	29				

5.   Result and Analysis

As one can see the highlighted cells in the image above, the F-value for sample and column i.e. factor 1(Music) and factor 2(Age) respectively, are higher than their F-critical values. This means that the factors have   a significant effect on the results of the students and thus we can reject the null hypothesis for the factors. Also the F-value for interaction effect is quite less than its F-critical value, so we can conclude that   music and age did not have any combined effect on the population.

6.   Conclusion

In many statistical applications in business administration, psychology, social science, and the natural sciences we need to compare more than two groups. For hypothesis testing more than two population means scientists have developed ANOVA method.

The ANOVA test procedure compares the variation in observations between samples (sum of squares for groups, SSC) to the variation within samples (sum of squares for error, SSE). The ANOVA F-test rejects the null hypothesis that the mean responses are equal in all groups if SSC is large relative to SSE.

The analysis of variance assumes that the observations are normally and independently distributed with the same variance for each treatment or factor level. If the normality assumption of the one-way ANOVA F-test is not met, we can use the Kruskal-Wallis rank test.

References

[1] Aczel, A.D., *Complete Business Statistics*, Irwin, 1989.

[2] Brown, M., Forsythe, A., "Robust tests for the equality of variances," *Journal of the American Statistical Association*, 364-367. 1974.

[3] Montgomery, D.C., Runger, G.C., *Applied Statistics and Probability for Engineers*, John Wiley & Sons, 2003.

[4] Ostertagová, E., *Applied Statistic* (in Slovak), Elfa, Košice, 2011.

[5] Parra-Frutos, I., "The behaviour of the modified Levene's test when data are not normally distributed," *Comput Stat*, Springer, 671-693. 2009.

[6] Rafter, J.A., Abell, M.L., Braselton, J.P., "Multiple Comparison Methods for Means," *SIAM Review*, 44 (2). 259-278. 2002.

[7] Rykov, V.V., Balakrishnan, N., Nikulin, M.S., *Mathematical and Statistical Models and Methods in Reliability*, Springer, 2010.

[8] Stephens, L.J., *Advanced Statistics demystified*, McGraw-Hill, 2004.

[9] Taylor, S., *Business Statistics*.www.palgrave.com.